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# SOME ASTROPHYSICAL CONSEQUENCES DUE TO THE EXISTENCE OF MAGNETIC MONOPOLES*

M. ORLANDINI

*Institute of Physics, University of Ferrara, Ferrara, Italy*

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This paper reviews the theory of the magnetic monopole as an astrophysical object. In particular we construct a model by which we are able to calculate the mass of the monopole by the unique assumption that the source of energy of quasars is due to monopole–antimonopole annihilation, without referring to particular GUTs. It results that the mass of the monopole must be $10^{16}$ GeV, i.e. the monopole predicted by the Georgi–Glashow model.

## 1. Introduction

Since 1931, when Dirac [1] introduced it for the first time, the magnetic monopole has been considered a mathematical object rather than a physical one, because the theory was not able to predict its physical properties and so the experimentalists had no clues to reveal it.

This situation was unchanged till 1974, when 't Hooft [2] and Polyakov [3] demonstrated the "necessity" of the existence of the magnetic monopole in the framework of a theory that predicts the unification of three of the four fundamental forces: the electromagnetic, the weak and the strong interactions.

The key point of the 't Hooft–Polyakov theory is that it is able to calculate the mass of the monopole in terms of free parameters of the Grand Unification Theories (GUTs). From this calculation it results that the monopole has a mass in the range of $10^4$–$10^{19}$ GeV [4,5], according to different theories, and the standard model of Georgi–Glashow [6] predicts a value of $10^{16}$ GeV. Because of the value of the mass, the monopole cannot be produced in any particle accelerators: an object of such energy can be produced only in the very early Universe, immediately after the "Big Bang".

So the magnetic monopole becomes a cosmological problem: in fact the matching between Cosmology and Particle Physics leads to the birth of the "new cosmology", which describes the GUTs in the framework of Einstein's General Theory of Relativity. From this matching a double benefit occurs: the cosmologists can now extrapolate the Big Bang theory to times less than one second without making

*Present address: International School for Advanced Studies - Strada Costiera 11 - 34014 Trieste - Italy aprioristic assumptions (i.e. they can justify the initial conditions) while the theorists employ the Universe as a laboratory in which to search for the tests of their theories.

In Part 2 of this paper we will give a review of the theory of the magnetic monopole, both the classical and the GUT one. In Part 3 we will describe the inflationary Universe model; in Part 4 we will calculate the limits to the density and the flux of the monopoles from astrophysical observations; in Part 5 we will construct a model of energy production for quasars, from which we obtain, for the mass of the monopole, a value of $10^{16}$ GeV, i.e. the mass predicted from the standard model of grand unification, the Georgi–Glashow model. In this paper, except Part 5, we use units for which $\hbar = c = k$ (Boltzmann constant) $= 1$.

## 2. Theory of the magnetic monopole

More than fifty years ago Dirac [1] demonstrated that the quantization of the electric charge can be explained supposing the existence of at least one free magnetic charge: the magnetic monopole.

Dirac considered the monopole like a semi-infinite and infinitesimal thin solenoid; in fact if we consider a monopole of magnetic charge $g$, placed in the origin of a polar coordinate system, it will produce a magnetic field of strength

$$\bar{B} = g\frac{\hat{r}}{r^2}, \tag{1}$$

where $\hat{r}$ is the radial unit vector. In a suitable gauge the vector potential associated to this field has the form

$$\bar{A} = g\frac{1+\cos\vartheta}{\sin\vartheta}\frac{\hat{r}}{r}. \tag{2}$$

Although (1) is singular only for $r = 0$, (2) has a singularity also for $\vartheta = 0$. This "string" singularity does not depend on the chosen gauge: we will always find a string singularity from $r = 0$ to $r = \infty$.

Because the quantum mechanical equations of charged particles are expressed directly in terms of the vector potential, it is necessary to make the singularity unobservable.

So we consider the effect that a trip around the string produces on the wawe function of a particle with electric charge $-e$: generally the phase of the wave function will be modified because of the vector potential $\bar{A}$

$$\psi \to \psi' = \exp[-ie\int \bar{A}\cdot d\bar{x}]\psi. \tag{3}$$

Once the integral is computed, with the aid of (2), we obtain

$$\psi' = \exp[4\pi ige]\psi. \tag{4}$$

So the string singularity is not detectable if the magnetic charge of the monopole obeys the condition

$$g = \frac{n}{2e}, \tag{5}$$

when $n$ is an integer. The smallest magnetic charge is the Dirac magnetic charge $g_D$, the value of which is

$$g_D = 1/2e \sim 68.5e \sim 3.310^{-8} \text{ CGS units.} \tag{6}$$

Now we reverse the reasoning: we suppose there is a particle of electric charge $Q$ and zero magnetic charge. Because of the existence of the magnetic monopole we must have

$$\exp[4\pi i Q g_D] = 1 \tag{7}$$

and so

$$\frac{Q}{e} = \frac{1}{2eg_D} = m, \tag{8}$$

where $m$ is an integer. The existence of the magnetic monopole involves the quantization of the electric charge. Someone could object that, in the calculation of $g_D$, we should have used $-e/3$, the charge of the quark $d$, and not $-e$ because it should be considered the smallest electric charge. This is not true because of the quark confinement: in fact it implies that the colour charge is screened for a distance greater than some fermis so that the previous discussion is still right. The Dirac's theory cannot predict the mass of the monopole because, from the Maxwell equation

$$div(\bar{B}) = 0 \tag{9}$$

it follows that the electromagnetic mass, considering a pointlike monopole, must be divergent.

As we have seen, the existence of the magnetic monopole implies the quantization of electric charge; in 1974 the inverse has been demonstrated [2,3]: every model of grand unification in which the electric charge is quantized (and so the electromagnetic group U(1) is embedded in the group which is broken by the mechanism of spontaneous symmetry breaking [7], contains necessarily magnetic monopoles.

So if we apply the GUTs we can determine the mass and the radius of a monopole. For example [8] we consider the electroweak theory which has (for simplicity) as gauge bosons the massless photon $\gamma$ and the heavy $W^{\pm}$. As we have seen, (9) shows that the magnetic field of the electromagnetic interaction $\bar{B}^{\gamma}$ is everywhere divergent. Yet in this theory it is necessary to consider the "covariant" derivative [9] and not the usual one. So (9) becomes

$$\bar{D} \cdot \bar{B} = div(\bar{B}^{\gamma}) + ie\bar{A}^{+}\bar{B}^{-} - ie\bar{A}^{-}\bar{B}^{+} = 0, \tag{10}$$

where $\bar{A}^{\pm}$ and $\bar{B}^{\pm}$ are the fields associated with the bosons $W^{\pm}$. In this way (10) can be written in the form

$$div(\bar{B}^{\gamma}) = \rho, \tag{11}$$

where $\rho$ is the source term due to the gauge bosons:

$$\rho = ie(\bar{A}^{-}\bar{B}^{+} - \bar{A}^{+}\bar{B}^{-}). \tag{12}$$

Because it is possible for (12) to have a constant value, different from zero, in a region of the size of the Compton wavelength of the gauge bosons, this implies that the source is no longer pointlike and so there is no problem of divergence: the physical properties of monopoles are well-defined.

Actually it is not possible to use the electroweak theory because we need a theory which considers also the strong interaction. The Georgi–Glashow model, based on the symmetry breaking from the group SU(5) to the group SU(3) × SU(2) × U(1), predicts that the physical properties of monopoles are

$$r \approx \frac{1}{M_x} \approx 10^{-28}\text{cm}, \qquad m \approx \frac{M_x}{\alpha_{\text{GUT}}} \approx 10^{16}\text{GeV}, \tag{13}$$

where $M_x$ is the mass of one of the bosons that mediates the unified interactions and $\alpha_{\text{GUT}}$ the gauge coupling constant. As we can see (13) depends strictly from the particular GUT: in fact the mass of the monopole is predicted to be in the range from $10^4$ to $10^{16}$ GeV, according as we consider particular phase transitions [10] or we introduce also the gravitational interaction (Kaluza–Klein theories).

Because, as we have shown, the Maxwell equations break down at very short distance, the magnetic monopole must have an internal structure, which is as follows [11, 12, 13]

a) the core, with radius $r_c \sim 10^{-28}$ cm, in which there are virtual X bosons;
b) the electroweak region, with radius $r_{ew} \sim 10^{-16}$ cm, containing virtual bosons $W^{\pm}, Z^0$ and $\gamma$;
c) the confinement region, with radius $r_{cf} \sim 10^{-13}$ cm, where there are strongly interacting objects (quarks, antiquarks and gluons);
d) a fermion–antifermion condensate, with radius of about one fermi for hadrons and few fermis for leptons, in which the baryonic number is not conserved;
e) for distances greater than few fermis the monopole acts like a Dirac monopole, yielding a magnetic field $|\vec{B}| = g_D/r^2$. From these results it follows that the monopole can act as a catalyst in the proton decay process [11, 12]

$$\begin{aligned} M + p &\rightarrow M + e^{+} + \pi^{0}, \\ M + p &\rightarrow M + e^{+}, \end{aligned} \tag{14}$$

with cross-section of the order of $10^{-27}\text{cm}^2$. Consequently, we have that the $M - \bar{M}$ annihilation cross section is larger than the classical one [14]; in fact from (14) it follows that the reaction

$$M + \bar{M} \rightarrow \bar{p} + e^{+} + \pi^{0} \tag{15}$$

has the same cross section of the catalysis process, i.e. $\sigma \sim 10^{-26}$ cm$^2$.

## 3. The inflationary Universe

The standard cosmological theory is the so-called "Big Bang" theory [15]. Starting from the assumption of an isotropic and homogeneous Universe and so described by a Robertson–Walker metric [16]

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\phi^2)\right\}, \tag{16}$$

where $k$ assumes the values $-1$, 0, $+1$ for an open, flat or closed Universe, respectively, it describes the evolution of the scale factor $R(t)$ by means of the Einstein–Fridman equations

$$\left(\frac{dR}{dt}\right)^2 = \frac{8}{3}\pi G\rho R^2 - k, \tag{17}$$

$$\frac{d^2R}{dt^2} = -\frac{4}{3}\pi G(\rho + 3p)R. \tag{18}$$

This system of equations, plus the energy conservation, the state equation of matter and the additional assumption of an adiabatic expansion can explain a lot of observed features of our Universe: the red-shifted light from distant galaxies; the cosmic microwave background radiation and the abundance of the primordial elements [15,16]: Deuterium and Helium.

But if we extrapolate the model to times less than one second we cannot understand why the Universe is so uniform on a great scale (the "horizon problem") [17, 18]; why the density of the Universe is so close to the critical one (the "flatness problem") [19] and why the Universe is composed only of matter and not of antimatter (the "baryon number" problem) [20]. To solve these problems the cosmologists imposed ad hoc conditions, assumed as initial conditions. Among these the less acceptable was the assumption that the Universe had to be originally asymmetrical between baryons and antibaryons while it is extremely plausible that matter and antimatter should appear on the same footing.

Just this last problem has led to introduce theories that predict the non conservation of the baryonic number: the GUTs. The GUTs predict that for very high energies the fundamental interactions are unified, i.e. mediated by the same gauge boson. As the energy (and so the temperature) drops, the interactions decoupled by means of a mechanism called spontaneous symmetry breaking, which is driven by a potential which assumes different forms according as the particular type of GUT is taken into account. The result which we arrive in applying these theories to the Einstein–Fridman equations is that the scale factor $R(t)$ has, for a certain period, an exponential law of evolution ("inflation") [21]

$$R(t)\alpha\begin{cases} e^{Ht} & T \ll T_c, \\ t^{1/2} & T \gg T_c, \end{cases} \tag{19}$$

where $H = (8\pi G V_0/3)^{1/2}$, $G$ is the Newton constant and $V_0$ the vacuum value of the potential. For a temperature larger than that at which the decoupling occurs $T_c$ the scale factor resumes the power law predicted from the standard theory.

This theory solves in a very elegant way all the previous problems but a new problem comes in: during the phase transition that decouples the interactions some defects occur, defects that physically correspond to magnetic monopoles.

If we calculate the number density of these monopoles we obtain [14]

$$n \sim 10^{-10}\, T^3 \tag{20}$$

of the same order of the baryon one. This is naturally against the experimental observations because the monopole mass is much larger than the baryon one (of the order of $10^{16}$!). To solve the "magnetic monopole" problem it has been considered a particular GUT, the Coleman–Weinberg theory [22]; but using this theory we obtain a monopole density extremely low [23,24,25]

$$n \sim 10^{-220}\, T^3 \tag{21}$$

so the direct observation of monopole becomes practically impossible. As we have seen the theory is not able to predict the magnetic monopole density unambiguously. In the next Section we will show how to calculate limits on the flux of monopoles from astrophysical observations.

## 4. Experimental limits to monopole density

Because the detectable monopoles come from cosmic rays, they are influenced by gravitational fields and/or magnetic fields. We can give an order of magnitude to the velocity acquired by a monopole free falling in the gravitational field of our galaxy; in fact

$$\bar{v} \sim \left(\frac{GM_G}{R_G}\right)^{1/2} \sim 10^{-3}c, \tag{22}$$

where $M_G \sim 10^{11} M_\odot \sim 10^{44} g$ and $R_G \sim 10^{22}$ cm are the mass and the radius of our galaxy [26]. As we can see [22] is mass independent.

To compute the effect of the galactic magnetic field on a monopole we remember that our galaxy has a magnetic field of $10^{-6}$ gauss uniform on a scale $L \sim 10^{21}$ cm; the velocity acquired by the monopole is

$$\begin{aligned} v &= \left(\frac{2gBL}{m_M}\right)^{1/2} = \\ &= \left(\frac{BL}{em_M}\right)^{1/2} \sim 3\cdot 10^{-3}c \left(\frac{B}{3\cdot 10^{-6}\text{gauss}}\right)^{1/2} \left(\frac{L}{10^{21}\text{cm}}\right)^{1/2} \left(\frac{10^{16}\text{GeV}}{m_M}\right)^{1/2} \end{aligned} \tag{23}$$

for a monopole of magnetic charge $g_D = 1/2e$.

From (23) we see that for $m_M \geq 10^{17}$ GeV the velocity acquired in the gravitational field is larger than that acquired in the magnetic one, moreover, that it is possible to consider the monopole as a classical object for $m_M \geq 10^{11}$ GeV.

For the interactions between monopoles and condensed matter it has been found [27] that, for rather slow monopoles ($\beta \sim 10^{-2}$), the stopping power is of the order of $10^{11}$ cm; in this way the Earth is trasparent to their motion.

A limit on the monopole density is first of all set by the critical density of matter in the Universe, i.e. $\rho \leq 10^{-29}$ g/cm so that

$$n \leq 10^{-21}\mathrm{cm}^{-3} \left(\frac{10^{16}\mathrm{GeV}}{m_M}\right). \tag{24}$$

Combining this expression with (23), valid for $m_M \leq 10^{17}$ GeV, we obtain

$$nv \leq 10^{-2}\mathrm{m}^{-2}\mathrm{yr}^{-1} \left(\frac{10^{16}\mathrm{GeV}}{m_M}\right)^{3/2}, \quad m_M \leq 10^{17}\mathrm{GeV}. \tag{25}$$

For a monopole mass greater than $10^{17}$ GeV the gravitational interaction prevails on the electromagnetic one so that the monopoles will be confined in the galactic halo, increasing their density. In this case, assuming a mass of the galactic halo of $10^{12}\mathrm{M}_\odot$ and a velocity of $10^{-3}c$ we obtain

$$nv \leq 10^{2}\mathrm{m}^{-2}\mathrm{yr}^{-1} \left(\frac{10^{17}\mathrm{GeV}}{m_M}\right), \qquad m_M \geq 10^{17}\mathrm{GeV}. \tag{26}$$

Moreover, is possible to obtain limits on the monopole flux by indirect reasoning, like the persistence of the galactic magnetic field [28,29,30] or the study of the X emission from neutron stars [31,32]. We see in detail these arguments. If the galactic magnetic field is due to the presence of persistent currents [33] (because rot $(\bar{B}_{\mathrm{gal}}) \neq 0$) then the monopoles that move in our galaxy acquired energy at the field expense, reducing consequently the currents. The actual observation of the galactic magnetic field implies that the rate at which the energy is dissipated is smaller than the regeneration rate of the currents by dynamo mechanism, with a period $\tau$ of about $10^8$ years, so we have that

$$nv \leq 2 \cdot 10^{-4}\mathrm{m}^{-2}\mathrm{yr}^{-1} \left(\frac{B}{3 \cdot 10^{-6}\mathrm{gauss}}\right) \left(\frac{10^{8}\mathrm{yr}}{\tau}\right), \tag{27}$$

which is independent of the mass of the monopole. Actually, we have neglected the effects of the gravitational field and this assumption is valid only for monopoles of mass smaller than $10^{17}$ GeV. In fact the very massive monopoles "feel" less the influence of the galactic magnetic field and so they extract from it less energy; in this way we obtain a less stringent limit on their flux

$$nv \leq 2 \cdot 10^{-4}\mathrm{m}^{-2}\mathrm{yr}^{-1} \left(\frac{m_M}{10^{17}\mathrm{GeV}}\right) \left(\frac{B}{3 \cdot 10^{-6}\mathrm{gauss}}\right) \left(\frac{10^{8}\mathrm{yr}}{\tau}\right); m_M \geq 10^{17}\mathrm{GeV}. \tag{28}$$

Combining (28) with (26) we obtain a mass independent limit for the monopole flux (Parker limit)

$$nv \leq 10^{-1}\mathrm{m}^{-2}\mathrm{yr}^{-1}. \tag{29}$$

We can obtain another limit on the monopole flux from the observation of neutron stars. In fact because of their very high density the neutron stars capture all the monopoles which fall on them and if the number of monopoles present in the star at the moment of its formation is negligible, then the number $N$ of monopoles present in a neutron star will be proportional to their flux. Because it has been found [11,12] that the monopoles act as catalyst in the nucleon decay process with cross section of the same order of that of strong interaction (of the order of $10^{-27}\mathrm{cm}^2$), it follows that also a little number of monopoles can cause a neutron star to emit in the ultraviolet and X. From observation in these frequency ranges it is possible to obtain a limit for $\sigma N$ and so limit for $nv$:

$$nv \leq 10^{-12}\mathrm{m}^{-2}\mathrm{yr}^{-1}\left(\frac{10^{-26}\mathrm{cm}^2}{\sigma}\right) \tag{30}$$

in clear contrast with (29).

## 5. $M - \bar{M}$ annihilation as "power supply" of quasars

The unsuccessful research of the magnetic monopoles shows that their number is now very little, below the sensibility of the detectors. But the theory predicts them so a mechanism that reduces their number is necessary, for example the $M - \bar{M}$ annihilation process. The energy emitted in this process, because of the mass of the monopole, would be enormous so we should observe "signs" of annihilation in very old astrophysical objects and therefore very close to the Big Bang. A good candidate as a seat of the $M - \bar{M}$ annihilation process seems to be quasars, quasi stellar objects characterized by having the lines of their spectra red-shifted of 16% and more with respect to the wavelength observed on the Earth. Explaining this shift as due to Doppler effect, they would be very far from us and so very close to the Big Bang. Once their distance is known, it is possible to calculate their intrinsic (absolute) luminosity, which is of the order of $10^{39}$W almost independent of the frequency of observation.

The physical properties of quasars are summarized in Table I [34]. From the analysis of the physical characteristics of quasars we deduce that the thermonuclear reactions cannot occur because the density is too low. The observed energy might be produced gravitationally but in this case the radiation pressure would prevail on the pressure of the gas, preventing the collapse. So our idea has been to consider a quasar like a cloud of mass $\mathcal{M}$ and volume $V$ containing a number of monopoles and antimonopole, with the same density, so that their mass energy does not exceed the mass energy of the matter.

Because the major contribution to the mass energy of matter is due to protons, it will be

$$N_p E_p = 2 N_M E_M, \tag{31}$$

**Table I**

| Object | $z$ | $L_{OPT}$ | $L_{IR}$ | $L_{RADIO}$ |
|---|---|---|---|---|
| 3CR 273 | 0.158 | 1.5 $10^{46}$ | 6.7 $10^{46}$ | 5.0 $10^{44}$ |
| 1400+16 | 0.244 | 1.0 $10^{45}$ | 7.6 $10^{44}$ | |
| 3C 279 | 0.536 | 3.1 $10^{46}$ | | 2.3 $10^{45}$ |
| 3C 446 | 1.404 | 2.1 $10^{46}$ | 5.4 $10^{47}$ | 6.4 $10^{45}$ |
| PKS2126-15 | 3.270 | 1.0 $10^{48}$ | | |

The mass of a quasar is $M \approx 10^{41} - 10^{43}$ g
The radius of a quasar is $R \approx 10^{16}$ cm

where the factor 2 has been introduced to distinguish monopoles from antimonopoles. The number of protons in a quasar is

$$N_p = \mathcal{M}/m_p, \tag{32}$$

so that the monopole (antimonopole) number density, supposing their volume constant in time and equal to the volume of the quasar, is

$$n_M = \frac{N_M}{V} = \frac{\mathcal{M}}{2VM_M(g)}\text{cm}^{-3}, \tag{33}$$

where $M_M(g)$ stands for the mass of the monopole expressed in grams. Because of continuous annihilation we have that the number of monopoles (antimonopoles) will be a time-dependent quantity; in particular

$$\frac{d}{dt}n_M(t) = - < \sigma v > n_M^2(t), \tag{34}$$

where $\sigma$ is the annihilation cross section, of order of $10^{-27}\text{cm}^2$ and $v \sim 10^{-3}c$ is the monopole velocity. Integrating (34) with the condition that for $t = 0$ $n_M = n_{M_0}$ we have

$$n_M(t) = \frac{n_{M_0}}{1+ < \sigma v > n_{M_0} t}, \tag{35}$$

where $n_{M_0}$ is given by (33).

The luminosity $\mathcal{L}$ of a quasar will become function of its age with the following expression

$$\mathcal{L} = 2M_M(g)c^2 n_M^2(t) < \sigma v > V = \frac{\mathcal{M}^2 c^2}{2VM_M(g)} < \sigma v > \left[1 + \frac{< \sigma v > \mathcal{M}t}{2VM_M(g)}\right]^{-2}. \tag{36}$$

The mass-luminosity diagram is shown in Fig. 1; as we can see there is a maximum luminosity, corresponding to the limit case of a quasar of infinite mass, given by

$$\mathcal{L}_{\max} = \frac{2VM_M(g)c^2}{<\sigma v> t^2}. \tag{37}$$

The inverse function, i.e. $\mathcal{M} = \mathcal{M}(\mathcal{L})$, of (36) is

$$\mathcal{M} = \frac{2VM_M(g)}{(2VM_M(g) <\sigma v> /\mathcal{L})^{1/2} - <\sigma v> t}. \tag{38}$$

From this expression we see that to have $\mathcal{M} > 0$ it is necessary for the denominator to be positive, i.e.

$$2VM_M/\mathcal{L} > <\sigma v> t^2. \tag{39}$$

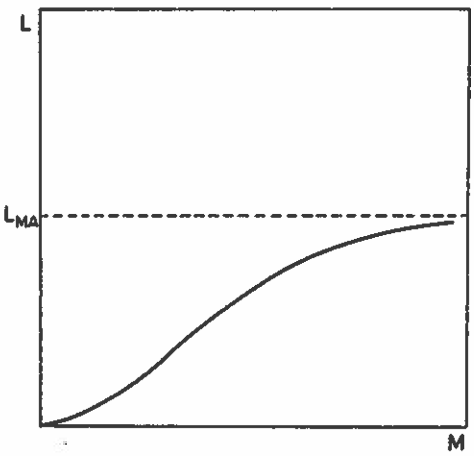

*Fig. 1*

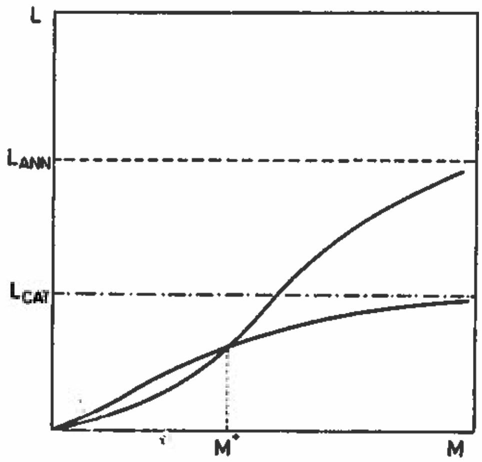

*Fig. 2*

This restriction on $\mathcal{L}$ implies also a restriction on the mass of the monopole. In fact if we consider $\mathcal{L} \sim \mathcal{L}_{\max} \sim 10^{39}$W we obtain Table II.

From this Table it is possible to see that the only monopole that allows for a luminosity smaller than $\mathcal{L}_{\max}$ and consistent with every observed red-shift is the monopole with mass of $10^{16}$ GeV. i.e. the monopole predicted by the Georgi–Glashow model.

Another mechanism which can increase the brightness of a quasar is the catalysis of the monopole on the proton decay. In this case the number density of the protons will vary in time with law

$$\frac{d}{dt}n_p(t) = - <\sigma v> n_M(t)n_p(t), \tag{40}$$

**Table II**

| $E_M$(GeV) | $z$ | $M(g)$ | $E_M$(GeV) | $z$ | $M(g)$ |
|---|---|---|---|---|---|
| $10^{16}$ | 3 | 7.83 $10^{42}$ | $10^{16}$ | 1 | 8.69 $10^{42}$ |
| $10^{15}$ | 3 | 2.80 $10^{42}$ | $10^{15}$ | 1 | 4.34 $10^{42}$ |
| $10^{14}$ | 3 | 1.53 $10^{42}$ | $10^{14}$ | 1 | −1.65 $10^{42}$ |
| $10^{13}$ | 3 | −3.78 $10^{41}$ | $10^{13}$ | 1 | −6.55 $10^{40}$ |
| $10^{4}$ | 3 | −1.45 $10^{32}$ | $10^{4}$ | 1 | −5.12 $10^{31}$ |
| $10^{16}$ | 2 | 8.07 $10^{42}$ | $10^{16}$ | 0.5 | 9.57 $10^{42}$ |
| $10^{15}$ | 2 | 3.13 $10^{42}$ | $10^{15}$ | 0.5 | 7.99 $10^{42}$ |
| $10^{14}$ | 2 | 3.53 $10^{42}$ | $10^{14}$ | 0.5 | −6.33 $10^{41}$ |
| $10^{13}$ | 2 | −1.57 $10^{41}$ | $10^{16}$ | 0.1 | 1.15 $10^{43}$ |
| $10^{4}$ | 2 | −9.42 $10^{31}$ | $10^{15}$ | 0.1 | −1.89 $10^{43}$ |

where the catalysis cross section is equal to the annihilation one. Developing the same kind of calculations done for the annihilation process we obtain that the luminosity produced by the catalysis process is

$$\mathcal{L}_{\text{cat}} = \frac{\mathcal{M}^2c^2}{VM_M(g)} < \sigma v > \left[1 + \frac{< \sigma v > \mathcal{M}t}{VM_M(g)}\right]^{-2}, \tag{41}$$

which compared with (36) shows that the two processes differ in the fact that in the annihilation there is $2V$ while in the catalysis there is only $V$. With the same arguments as the previous one we arrive at a constraint on the monopole mass, which must be necessarily $10^{16}$ GeV.

Now it is necessary to establish for which values of the physical parameters a process prevails on the other. In order that (36) is greater than (41) it has to be

$$\mathcal{M} > \frac{\sqrt{2}VM_M(g)}{< \sigma v > t} \equiv \mathcal{M}^*, \tag{42}$$

so for a certain age of the quasar the brightness due to annihilation prevails on that due to catalysis if $\mathcal{M} > \mathcal{M}^*$ (Fig. 2). From Table I we have, for a typical quasar

$$\mathcal{M}^* = 1.3 \cdot 10^{43}(1+z)^{3/2} \tag{43}$$

and so the observed luminosity is due to the catalysis process. This model can explain effectively the energy emitted from quasars, determining moreover the mass of the monopole, but it contains a "bug": from (35) we have that the time necessary to reduce the initial number of monopoles to half is much greater than the age of the Universe so it would be possible to observe them: of course it is not in agreement with the observational data. A way to solve this problem might be to assume that the volume occupied by the monopoles is a time-dependent quantity, but this is at the moment, not clear, yet.

## 6. Conclusion

The theoretical previsions about the number of monopoles that we should observe now are not in agreement, depending critically on the GUTs adopted. The mass itself of the monopole, in the framework of various theories, can vary widely: from $10^4$ to $10^{19}$ GeV.

The only experimental data about monopoles we have at our disposal is that their number is actually below the sensitivity of the detectors, i.e. the present number of monopoles is practically zero. But because the theory predicts their existence a mechanism is necessary which is able to reduce their number in an effective manner; such a mechanism might be the monopole–antimonopole annihilation.

Because of the enormous mass of the monopole, the energy released in the process is very high and so it seems natural to us to think of their annihilation as taking place in quasars, which are very old and bright heavenly objects with not yet known source of energy.

The interest of this model resides in the fact that it can explain the energy emitted by the quasars and from the measurement of their luminosity it is obtained that the ONLY monopole that can produce such an energy must be the $10^{16}$ GeV monopole, i.e. the Georgi–Glashow monopole. One must point out that this result is obtained by the unique assumption that the source of energy of quasars is the $M - \bar{M}$ annihilation and so without referring to particular GUTs.

Moreover, if we suppose that the monopoles, inside the quasars, tend to gather towards the centre, then we have that the annihilation time is of the same order of magnitude of the age of the Universe and the volume occupied by them is very small: this makes them practically undetectable.

## References

1. P.A.M. Dirac, Proc. Roy. Soc., *A133*, 60, 1931.
2. G. 't Hooft, Nucl. Phys. *B79*, 276, 1974; *B105*, 538, 1976.
3. A.M. Polyakov, JETP Lett., *20*, 194, 1974.
4. P. Goddard and D. Olive, Rep. Progr. Phys., *41*, 1357, 1978.
5. J.P. Preskill, Ann. Rev. Nucl. Part. Sc. *34*, 1984.
6. H. Georgi and S.L. Glashow, Phys. Rev. Lett., *33*, 438, 1974.
7. P. Langacker, Phys. Rep., *72C*, 185, 1981.
8. C.G. Callan, in Beyond the standard model, pp. 412 - 413, Frontieres, Gif sur Yvette, 1983.
9. J. Leite-Lopes, Gauge field theories, an introduction, Pergamon Press, London, 1983.
10. A.H. Guth and E. Weinberg, Phys. Rev.,*D23*, 876, 1981.
11. C.G. Callan, Phys. Rev., *D25*, 2141, 1982; *D26*, 2058, 1982.
12. V. Rubakov, JETP Lett., *33*, 644, 1982.
13. G. Giacomelli, CERN Preprint EP/84 - 17, 1984.
14. J.P. Preskill, Phys. Rev. Lett., *43*, 1365, 1979.
15. Ya.B. Zel'dovich and I.D. Novikov: Structure and evolution of the Universe, Riuniti, Roma, 1982.
16. S. Weinberg, Gravitation and cosmology, Wiley, New York, 1972.
17. W. Rindler, Mon. Not. R. Astron. Soc., *116*, 663, 1956.
18. C.W. Misner, K.S. Thorne and J.A. Wheeler, Gravitation, Freeman, S. Francisco, 1973.

19. R.H. Dicke and P.J. Peebles, General relativity: an Einstein centenary survey, Cambridge University Press, London, 1979.
20. F. Wilczek, Scientific American (Italian edition), *150*, 28, 1981.
21. A.H. Guth, Phys. Rev., *D23*, 347, 1981.
22. S. Coleman and E. Weinberg, Phys. Rev., *D7*, 1888, 1973.
23. A.D. Linde, Phys. Lett., *B108*, 389, 1982; *B114*, 431, 1982.
24. A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett., *48*, 1220, 1982.
25. J.P. Preskill, in The very early Universe, Cambridge University Press, London 1983.
26. L. Gratton, Introduzione all'astrofisica, Zanichelli, Bologna, 1982.
27. S.P. Ahlen and K. Kinoshita, Phys. Rev., *D26*, 2347, 1982.
28. E.N. Parker, Ap. J. *160*, 383, 1970; *163*, 255, 1971; *166*, 295, 1971.
29. S.A. Bludman and M.A. Ruderman, Phys. Rev. Lett., *36*, 840, 1976.
30. M.S. Turner, E.N. Parker and T.J. Bogdan, Phys. Rev., *D26*, 1296, 1982.
31. S.Dimopoulos, J.P. Preskill and F. Wilczek, Phys. Lett., *B119*, 320, 1982.
32. E.W. Kolb, S. A. Colgate and J. A. Harvey, Phys. Rev. Lett., *49*, 1373, 1982.
33. E.N. Parker, Cosmological magnetic fields, Clarendon, Oxford, 1979.
34. G. Tanzella-Nitti, Coelum, *52*, 1, 1984.